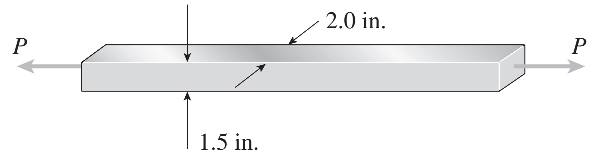
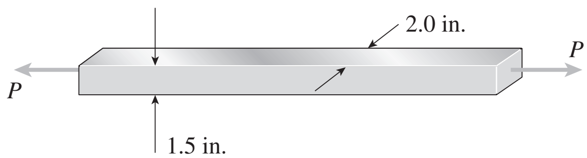
Stresses on Inclined Sections

Problem 2.6-1 A steel bar of rectangular cross section (1.5 in. × 2.0 in.) carries a tensile load P (see figure). The allowable stresses in tension and shear are 15,000 psi and 7,000 psi, respectively.

Determine the maximum permissible load P_{max}.



Solution 2.6-1 **Rectangular bar in tension**



$A = 1.5 \text{ in.} \times 2.0 \text{ in.}$

$= 3.0 \text{ in.}^2$

Maximum Normal Stress:

$$\sigma_x = \frac{P}{A}$$

Maximum shear stress: $\tau_{max} = \dfrac{\sigma_x}{2} = \dfrac{P}{2A}$

$\sigma_{allow} = 15{,}000 \text{ psi} \quad \tau_{allow} = 7{,}000 \text{ psi}$

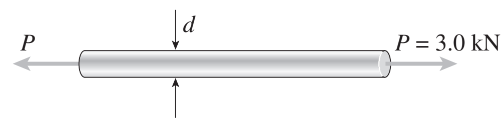
Because τ_{allow} is less than one-half of σ_{allow}, the shear stress governs.

$P_{max} = 2\tau_{allow} A = 2(7{,}000 \text{ psi})(3.0 \text{ in.}^2)$

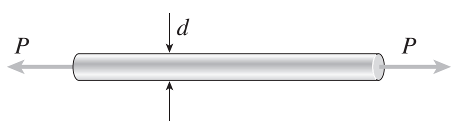
$= 42{,}000 \text{ lb}$ ←

Problem 2.6-2 A circular steel rod of diameter d is subjected to a tensile force $P = 3.0$ kN (see figure). The allowable stresses in tension and shear are 120 MPa and 50 MPa, respectively.

What is the minimum permissible diameter d_{min} of the rod?



Solution 2.6-2 **Steel rod in tension**



$P = 3.0 \text{ kN} \quad A = \dfrac{\pi d^2}{4}$

Maximum normal stress: $\sigma_x = \dfrac{P}{A}$

Maximum shear stress: $\tau_{max} = \dfrac{\sigma_x}{2} = \dfrac{P}{2A}$

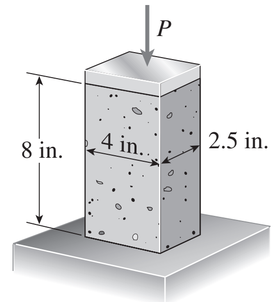
$\sigma_{allow} = 120 \text{ MPa} \qquad \tau_{allow} = 50 \text{ MPa}$

Because τ_{allow} is less than one-half of σ_{allow}, the shear stress governs.

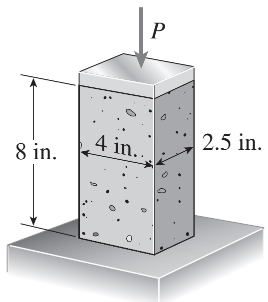
$$\tau_{max} = \frac{P}{2A} \quad \text{or} \quad 50 \text{ MPa} = \frac{3.0 \text{ kN}}{(2)\left(\dfrac{\pi d^2}{4}\right)}$$

Solve for d: $d_{min} = 6.18$ mm ←

Problem 2.6-3 A standard brick (dimensions 8 in. × 4 in. × 2.5 in.) is compressed lengthwise by a force P, as shown in the figure. If the ultimate shear stress for brick is 1200 psi and the ultimate compressive stress is 3600 psi, what force P_{max} is required to break the brick?



Solution 2.6-3 Standard brick in compression



$A = 2.5 \text{ in.} \times 4.0 \text{ in.} = 10.0 \text{ in.}^2$

Maximum normal stress:

$$\sigma_x = \frac{P}{A}$$

Maximum shear stress:

$$\tau_{max} = \frac{\sigma_x}{2} = \frac{P}{2A}$$

$\sigma_{ult} = 3600$ psi $\qquad \tau_{ult} = 1200$ psi

Because τ_{ult} is less than one-half of σ_{ult}, the shear stress governs.

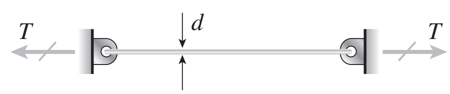
$$\tau_{max} = \frac{P}{2A} \quad \text{or} \quad P_{max} = 2A\tau_{ult}$$

$$P_{max} = 2(10.0 \text{ in.}^2)(1200 \text{ psi})$$

$$= 24{,}000 \text{ lb} \quad \longleftarrow$$

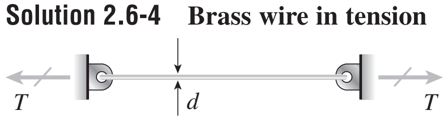
Problem 2.6-4 A brass wire of diameter $d = 2.42$ mm is stretched tightly between rigid supports so that the tensile force is $T = 92$ N (see figure).

What is the maximum permissible temperature drop ΔT if the allowable shear stress in the wire is 60 MPa? (The coefficient of thermal expansion for the wire is 20×10^{-6}/°C and the modulus of elasticity is 100 GPa.)



Probs. 2.6-4 and 2.6-5

Solution 2.6-4 Brass wire in tension



$d = 2.42$ mm

$$A = \frac{\pi d^2}{4} = 4.60 \text{ mm}^2$$

$\alpha = 20 \times 10^{-6}$/°C $\quad E = 100$ GPa $\quad \tau_{allow} = 60$ MPa

Initial tensile force: $T = 92$ N

Stress due to initial tension: $\sigma_x = \dfrac{T}{A}$

Stress due to temperature drop: $\sigma_x = E\alpha(\Delta T)$

(see Eq. 2-18 of Section 2.5)

Total stress: $\sigma_x = \dfrac{T}{A} + E\alpha(\Delta T)$

MAXIMUM SHEAR STRESS

$$\tau_{max} = \frac{\sigma_x}{2} = \frac{1}{2}\left[\frac{T}{A} + E\alpha(\Delta T)\right]$$

Solve for temperature drop ΔT:

$$\Delta T = \frac{2\tau_{max} - T/A}{E\alpha} \qquad \tau_{max} = \tau_{allow}$$

SUBSTITUTE NUMERICAL VALUES:

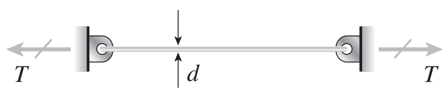
$$\Delta T = \frac{2(60 \text{ MPa}) - (92 \text{ N})/(4.60 \text{ mm}^2)}{(100 \text{ GPa})(20 \times 10^{-6}/°\text{C})}$$

$$= \frac{120 \text{ MPa} - 20 \text{ MPa}}{2 \text{ MPa/°C}} = 50°\text{C} \quad \longleftarrow$$

Problem 2.6-5 A brass wire of diameter $d = 1/16$ in. is stretched between rigid supports with an initial tension T of 32 lb (see figure).

(a) If the temperature is lowered by 50°F, what is the maximum shear stress τ_{max} in the wire?
(b) If the allowable shear stress is 10,000 psi, what is the maximum permissible temperature drop? (Assume that the coefficient of thermal expansion is 10.6×10^{-6}/°F and the modulus of elasticity is 15×10^6 psi.)

Solution 2.6-5 **Brass wire in tension**



$$d = \frac{1}{16} \text{ in.}$$

$$A = \frac{\pi d^2}{4} = 0.003068 \text{ in.}^2$$

$$\alpha = 10.6 \times 10^{-6}/°\text{F}$$

$$E = 15 \times 10^6 \text{ psi}$$

Initial tensile force: $T = 32$ lb

Stress due to initial tension: $\sigma_x = \frac{T}{A}$

Stress due to temperature drop: $\sigma_x = E\alpha(\Delta T)$

(see Eq. 2-18 of Section 2.5)

Total stress: $\sigma_x = \frac{T}{A} + E\alpha(\Delta T)$

(a) Maximum shear stress when temperature drops 50°F

$$\tau_{max} = \frac{\sigma_x}{2} = \frac{1}{2}\left[\frac{T}{A} + E\alpha(\Delta T)\right] \quad \text{(Eq. 1)}$$

Substitute numerical values:

$\tau_{max} = 9{,}190$ psi ←

(b) Maximum permissible temperature drop if $\tau_{allow} = 10{,}000$ psi ←

Solve Eq. (1) for ΔT:

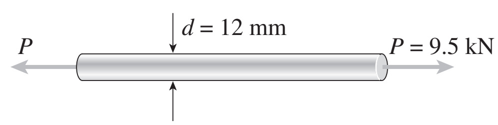
$$\Delta T = \frac{2\tau_{max} - T/A}{E\alpha} \qquad \tau_{max} = \tau_{allow}$$

Substitute numerical values:

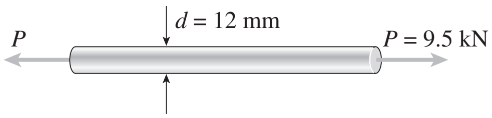
$\Delta T = 60.2°$F ←

Problem 2.6-6 A steel bar with diameter $d = 12$ mm is subjected to a tensile load $P = 9.5$ kN (see figure).

(a) What is the maximum normal stress σ_{max} in the bar?
(b) What is the maximum shear stress τ_{max}?
(c) Draw a stress element oriented at 45° to the axis of the bar and show all stresses acting on the faces of this element.



Solution 2.6-6 **Steel bar in tension**



$P = 9.5$ kN

(a) MAXIMUM NORMAL STRESS

$$\sigma_x = \frac{P}{A} = \frac{9.5\text{ kN}}{\frac{\pi}{4}(12\text{ mm})^2} = 84.0\text{ MPa}$$

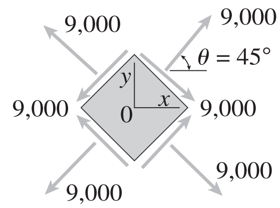
$\sigma_{max} = 84.0$ MPa ⟵

(b) MAXIMUM SHEAR STRESS

The maximum shear stress is on a 45° plane and equals $\sigma_x/2$.

$$\tau_{max} = \frac{\sigma_x}{2} = 42.0\text{ MPa}$$ ⟵

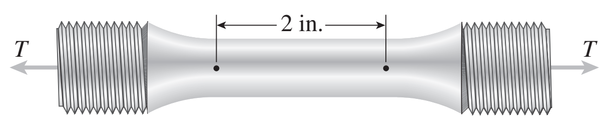
(c) STRESS ELEMENT AT $\theta = 45°$



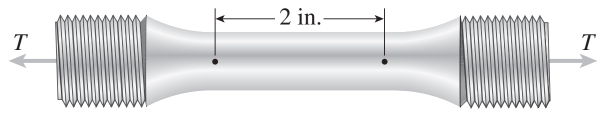
NOTE: All stresses have units of MPa.

Problem 2.6-7 During a tension test of a mild-steel specimen (see figure), the extensometer shows an elongation of 0.00120 in. with a gage length of 2 in. Assume that the steel is stressed below the proportional limit and that the modulus of elasticity $E = 30 \times 10^6$ psi.

(a) What is the maximum normal stress σ_{max} in the specimen?
(b) What is the maximum shear stress τ_{max}?
(c) Draw a stress element oriented at an angle of 45° to the axis of the bar and show all stresses acting on the faces of this element.



Solution 2.6-7 Tension test



Elongation: $\delta = 0.00120$ in.

(2 in. gage length)

Strain: $\varepsilon = \dfrac{\delta}{L} = \dfrac{0.00120 \text{ in.}}{2 \text{ in.}} = 0.00060$

Hooke's law : $\sigma_x = E\varepsilon = (30 \times 10^6 \text{ psi})(0.00060)$

$= 18{,}000$ psi

(a) MAXIMUM NORMAL STRESS

σ_x is the maximum normal stress.

$\sigma_{\max} = 18{,}000$ psi ←

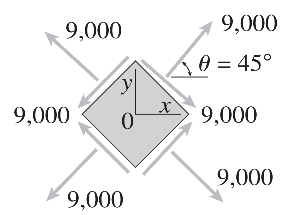
(b) MAXIMUM SHEAR STRESS

The maximum shear stress is on a 45° plane and equals $\sigma_x/2$.

$\tau_{\max} = \dfrac{\sigma_x}{2} = 9{,}000$ psi ←

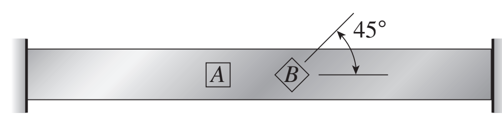
(c) STRESS ELEMENT AT $\theta = 45°$

NOTE: All stresses have units of psi.

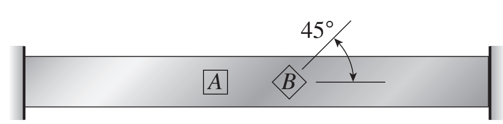


Problem 2.6-8 A copper bar with a rectangular cross section is held without stress between rigid supports (see figure). Subsequently, the temperature of the bar is raised 50°C.

Determine the stresses on all faces of the elements A and B, and show these stresses on sketches of the elements. (Assume $\alpha = 17.5 \times 10^{-6}$/°C and $E = 120$ GPa.)



Solution 2.6-8 Copper bar with rigid supports



$\Delta T = 50°\text{C}$ (Increase)

$\alpha = 17.5 \times 10^{-6}$/°C

$E = 120$ GPa

STRESS DUE TO TEMPERATURE INCREASE

$\sigma_x = E\alpha\,(\Delta T)$ (See Eq. 2-18 of Section 2.5)

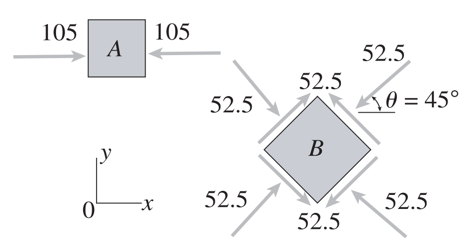
$= 105$ MPa (Compression)

MAXIMUM SHEAR STRESS

$\tau_{\max} = \dfrac{\sigma_x}{2}$

$= 52.5$ MPa

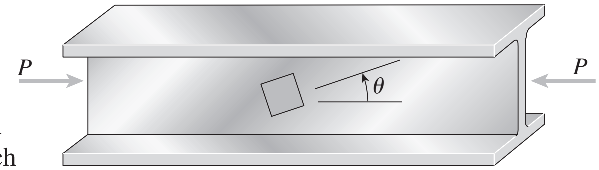
STRESSES ON ELEMENTS A AND B



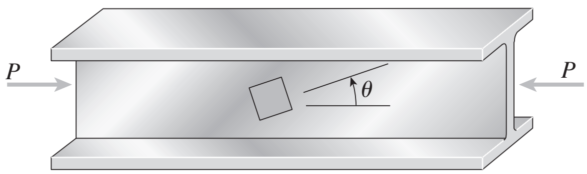
NOTE: All stresses have units of MPa.

Problem 2.6-9 A compression member in a bridge truss is fabricated from a wide-flange steel section (see figure). The cross-sectional area $A = 7.5$ in.^{2} and the axial load $P = 90$ k.

Determine the normal and shear stresses acting on all faces of stress elements located in the web of the beam and oriented at (a) an angle $\theta = 0°$, (b) an angle $\theta = 30°$, and (c) an angle $\theta = 45°$. In each case, show the stresses on a sketch of a properly oriented element.



Solution 2.6-9 **Truss member in compression**



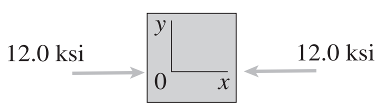
$P = 90$ k

$A = 7.5$ in.^{2}

$$\sigma_x = -\frac{P}{A} = -\frac{90 \text{ k}}{7.5 \text{ in.}^2}$$

$= -12.0$ ksi (Compression)

(a) $\theta = 0°$



(b) $\theta = 30°$

Use Eqs. (2-29a) and (2-29b):

$\sigma_\theta = \sigma_x \cos^2\theta = (-12.0 \text{ ksi})(\cos 30°)^2$

$= -9.0$ ksi

$\tau_\theta = -\sigma_x \sin\theta \cos\theta = -(-12.0 \text{ ksi})(\sin 30°)(\cos 30°)$

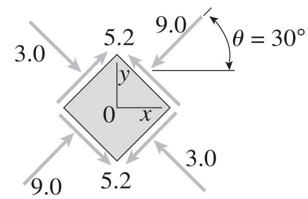
$= 5.2$ ksi

$\theta = 30° + 90° = 120°$

$\sigma_\theta = \sigma_x \cos^2\theta = (-12.0 \text{ ksi})(\cos 120°)^2 = -3.0$ ksi

$\tau_\theta = -\sigma_x \sin\theta \cos\theta = -(-12.0 \text{ ksi})(\sin 120°)(\cos 120°)$

$= -5.2$ ksi



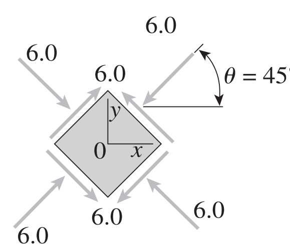
NOTE: All stresses have units of ksi.

(c) $\theta = 45°$

$\sigma_\theta = \sigma_x \cos^2\theta = (-12.0 \text{ ksi})(\cos 45°)^2 = -6.0$ ksi

$\tau_\theta = -\sigma_x \sin\theta \cos\theta = -(-12.0 \text{ ksi})(\sin 45°)(\cos 45°)$

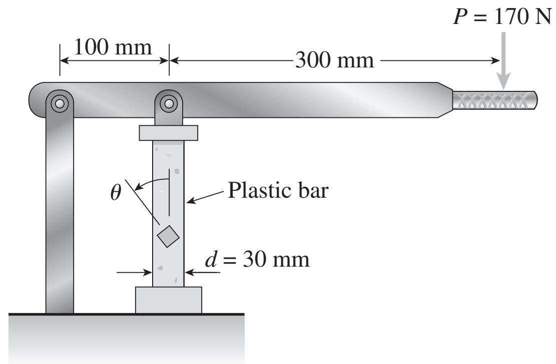
$= 6.0$ ksi



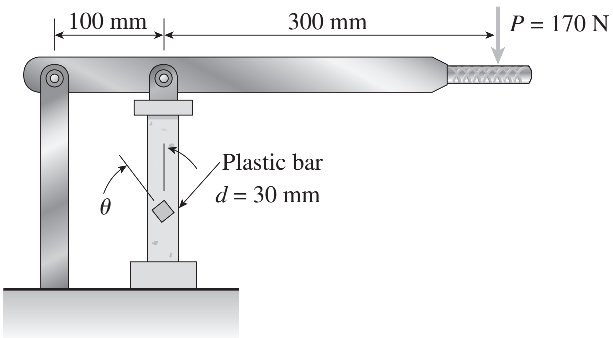
NOTE: All stresses have units of ksi.

Problem 2.6-10 A plastic bar of diameter $d = 30$ mm is compressed in a testing device by a force $P = 170$ N applied as shown in the figure.

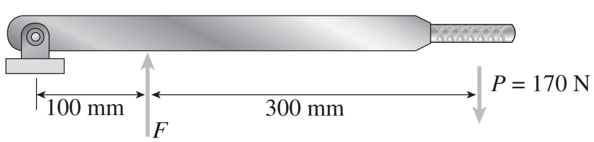
Determine the normal and shear stresses acting on all faces of stress elements oriented at (a) an angle $\theta = 0°$, (b) an angle $\theta = 22.5°$, and (c) an angle $\theta = 45°$. In each case, show the stresses on a sketch of a properly oriented element.



Solution 2.6-10 Plastic bar in compression



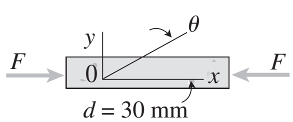
FREE-BODY DIAGRAM



F = Compressive force in plastic bar

$F = 4P = 4(170 \text{ N}) = 680 \text{ N}$

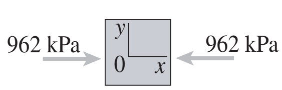
PLASTIC BAR (ROTATED TO THE HORIZONTAL)



$$\sigma_x = -\frac{F}{A} = -\frac{680 \text{ N}}{\frac{\pi}{4}(30 \text{ mm})^2}$$

$$= -962.0 \text{ kPa (Compression)}$$

(a) $\theta = 0°$



(b) $\theta = 22.5°$

Use Eqs. (2-29a) and (2-29b)

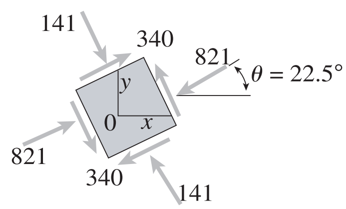
$$\sigma_\theta = \sigma_x \cos^2\theta = (-962.0 \text{ kPa})(\cos 22.5°)^2$$
$$= -821 \text{ kPa}$$

$$\tau_\theta = -\sigma_x \sin\theta \cos\theta$$
$$= -(-962.0 \text{ kPa})(\sin 22.5°)(\cos 22.5°)$$
$$= 340 \text{ kPa}$$

$$\theta = 22.5° + 90° = 112.5°$$

$$\sigma_\theta = \sigma_x \cos^2\theta = (-962.0 \text{ kPa})(\cos 112.5°)^2$$
$$= -141 \text{ kPa}$$

$$\tau_\theta = -\sigma_x \sin\theta \cos\theta$$
$$= -(-962.0 \text{ kPa})(\sin 112.5°)(\cos 112.5°)$$
$$= -340 \text{ kPa}$$

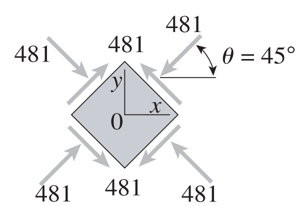


NOTE: All stresses have units of kPa.

(c) $\theta = 45°$

$$\sigma_\theta = \sigma_x \cos^2\theta = (-962.0 \text{ kPa})(\cos 45°)^2$$
$$= -481 \text{ kPa}$$

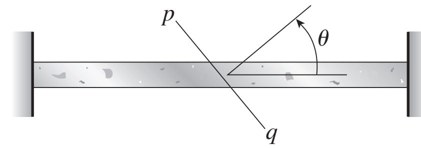
$$\tau_\theta = -\sigma_x \sin\theta \cos\theta$$
$$= -(-962.0 \text{ kPa})(\sin 45°)(\cos 45°) = 481 \text{ kPa}$$



NOTE: All stresses have units of kPa.

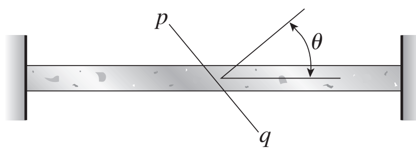
Problem 2.6-11 A plastic bar fits snugly between rigid supports at room temperature (68°F) but with no initial stress (see figure). When the temperature of the bar is raised to 160°F, the compressive stress on an inclined plane *pq* becomes 1700 psi.

(a) What is the shear stress on plane *pq*? (Assume $\alpha = 60 \times 10^{-6}$/°F and $E = 450 \times 10^3$ psi.)
(b) Draw a stress element oriented to plane *pq* and show the stresses acting on all faces of this element.



Probs. 2.6-11 and 2.6-12

Solution 2.6-11 **Plastic bar between rigid supports**



$\alpha = 60 \times 10^{-6}$/°F $\quad E = 450 \times 10^3$ psi

Temperature increase:

$\Delta T = 160°\text{F} - 68°\text{F} = 92°\text{F}$

NORMAL STRESS σ_x IN THE BAR

$\sigma_x = -E\alpha(\Delta T)$ (See Eq. 2-18 in Section 2.5)

$\sigma_x = -(450 \times 10^3 \text{ psi})(60 \times 10^{-6}/°\text{F})(92°\text{F})$

$= -2484$ psi (Compression)

ANGLE θ TO PLANE *pq*

$\sigma_\theta = \sigma_x \cos^2\theta$ For plane *pq*: $\sigma_\theta = -1700$ psi

Therefore, $-1700 \text{ psi} = (-2484 \text{ psi})(\cos^2\theta)$

$$\cos^2\theta = \frac{-1700 \text{ psi}}{-2484 \text{ psi}} = 0.6844$$

$\cos\theta = 0.8273 \quad \theta = 34.18°$

(a) SHEAR STRESS ON PLANE *pq*

$\tau_\theta = -\sigma_x \sin\theta \cos\theta$

$= -(-2484 \text{ psi})(\sin 34.18°)(\cos 34.18°)$

$= 1150$ psi (Counter clockwise) ⟵

(b) STRESS ELEMENT ORIENTED TO PLANE *pq*

$\theta = 34.18° \quad \sigma_\theta = -1700 \text{ psi} \quad \tau_\theta = 1150 \text{ psi}$

$\theta = 34.18° + 90° = 124.18°$

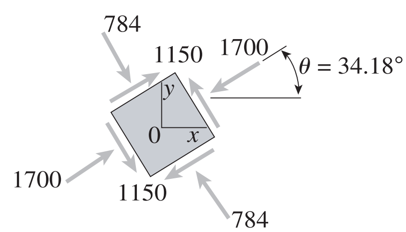
$\sigma_\theta = \sigma_x \cos^2\theta = (-2484 \text{ psi})(\cos 124.18°)^2$

$= -784$ psi

$\tau_\theta = -\sigma_x \sin\theta \cos\theta$

$= -(-2484 \text{ psi})(\sin 124.18°)(\cos 124.18°)$

$= -1150$ psi

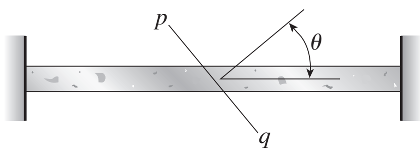


NOTE: All stresses have units of psi.

Problem 2.6-12 A copper bar is held snugly (but without any initial stress) between rigid supports (see figure). The allowable stresses on the inclined plane *pq*, for which $\theta = 55°$, are specified as 60 MPa in compression and 30 MPa in shear.

(a) What is the maximum permissible temperature rise ΔT if the allowable stresses on plane *pq* are not to be exceeded? (Assume $\alpha = 17 \times 10^{-6}/°C$ and $E = 120$ GPa.)
(b) If the temperature increases by the maximum permissible amount, what are the stresses on plane *pq*?

Solution 2.6-12 **Copper bar between rigid supports**



$\alpha = 17 \times 10^{-6}/°C$

$E = 120$ GPa

Plane *pq*: $\theta = 55°$

Allowable stresses on plane *pq*:

$\sigma_{allow} = 60$ MPa (Compression)

$\tau_{allow} = 30$ MPa (Shear)

(a) MAXIMUM PERMISSIBLE TEMPERATURE RISE ΔT

$\sigma_\theta = \sigma_x \cos^2\theta \quad -60 \text{ MPa} = \sigma_x (\cos 55°)^2$

$\sigma_x = -182.4$ MPa

$\tau_\theta = -\sigma_x \sin\theta \cos\theta$

$30 \text{ MPa} = -\sigma_x (\sin 55°)(\cos 55°)$

$\sigma_x = -63.85$ MPa

Shear stress governs. $\sigma_x = -63.85$ MPa

Due to temperature increase ΔT:

$\sigma_x = -E\alpha(\Delta T)$ (See Eq. 2-18 in Section 2.5)

$-63.85 \text{ MPa} = -(120 \text{ GPa})(17 \times 10^{-6}/°C)(\Delta T)$

$\Delta T = 31.3°C$ ←

(b) STRESSES ON PLANE *pq*

$\sigma_x = -63.85$ MPa

$\sigma_\theta = \sigma_x \cos^2\theta = (-63.85 \text{ MPa})(\cos 55°)^2$

$= -21.0$ MPa (Compression) ←

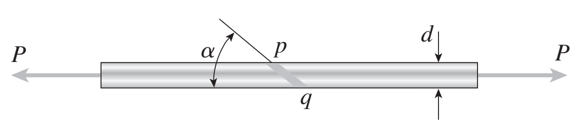
$\tau_\theta = -\sigma_x \sin\theta \cos\theta$

$= -(-63.85 \text{ MPa})(\sin 55°)(\cos 55°)$

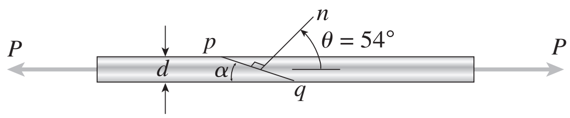
$= 30.0$ MPa (Counter clockwise) ←

Problem 2.6-13 A circular brass bar of diameter *d* is composed of two segments brazed together on a plane *pq* making an angle $\alpha = 36°$ with the axis of the bar (see figure). The allowable stresses in the brass are 13,500 psi in tension and 6500 psi in shear. On the brazed joint, the allowable stresses are 6000 psi in tension and 3000 psi in shear.

If the bar must resist a tensile force $P = 6000$ lb, what is the minimum required diameter d_{min} of the bar?



Solution 2.6-13 Brass bar in tension



$\alpha = 36°$

$\theta = 90° - \alpha = 54°$

$P = 6000 \text{ lb}$

$$A = \frac{\pi d^2}{4}$$

STRESS σ_x BASED UPON ALLOWABLE STRESSES IN THE BRASS

Tensile stress ($\theta = 0°$): $\sigma_{allow} = 13{,}500 \text{ psi}$

$\sigma_x = 13{,}500 \text{ psi}$ (1)

Shear stress ($\theta = 45°$): $\tau_{allow} = 6500 \text{ psi}$

$$\tau_{max} = \frac{\sigma_x}{2}$$

$$\sigma_x = 2\,\tau_{allow} = 13{,}000 \text{ psi} \qquad (2)$$

STRESS σ_x BASED UPON ALLOWABLE STRESSES ON THE BRAZED JOINT ($\theta = 54°$)

$\sigma_{allow} = 6000 \text{ psi (tension)}$

$\tau_{allow} = 3000 \text{ psi (shear)}$

Tensile stress: $\sigma_\theta = \sigma_x \cos^2\theta$

$$\sigma_x = \frac{\sigma_{allow}}{\cos^2\theta} = \frac{6000 \text{ psi}}{(\cos 54°)^2} = 17{,}370 \text{ psi} \qquad (3)$$

Shear stress: $\tau_\theta = -\sigma_x \sin\theta \cos\theta$

$$\sigma_x = \left|\frac{\tau_{allow}}{\sin\theta\cos\theta}\right| = \frac{3{,}000 \text{ psi}}{(\sin 54°)(\cos 54°)} = 6{,}310 \text{ psi} \qquad (4)$$

ALLOWABLE STRESS

Compare (1), (2), (3), and (4).

Shear stress on the brazed joint governs.

$\sigma_x = 6310 \text{ psi}$

DIAMETER OF BAR

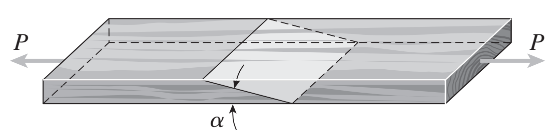
$$A = \frac{P}{\sigma_x} = \frac{6000 \text{ lb}}{6310 \text{ psi}} = 0.951 \text{ in.}^2$$

$$A = \frac{\pi d^2}{4} \quad d^2 = \frac{4A}{\pi} \quad d_{min} = \sqrt{\frac{4A}{\pi}}$$

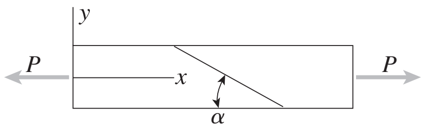
$d_{min} = 1.10 \text{ in.}$ ←

Problem 2.6-14 Two boards are joined by gluing along a scarf joint, as shown in the figure. For purposes of cutting and gluing, the angle α between the plane of the joint and the faces of the boards must be between 10° and 40°. Under a tensile load P, the normal stress in the boards is 4.9 MPa.

(a) What are the normal and shear stresses acting on the glued joint if $\alpha = 20°$?
(b) If the allowable shear stress on the joint is 2.25 MPa, what is the largest permissible value of the angle α?
(c) For what angle α will the shear stress on the glued joint be numerically equal to twice the normal stress on the joint?



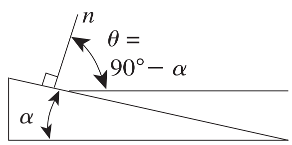
Solution 2.6-14 **Two boards joined by a scarf joint**



$10° \le \alpha \le 40°$

Due to load P: $\sigma_x = 4.9$ MPa

(a) Stresses on joint when $\alpha = 20°$



$$\theta = 90° - \alpha = 70°$$

$$\sigma_\theta = \sigma_x \cos^2\theta = (4.9 \text{ MPa})(\cos 70°)^2$$
$$= 0.57 \text{ MPa} \longleftarrow$$

$$\tau_\theta = -\sigma_x \sin\theta \cos\theta$$
$$= (-4.9 \text{ MPa})(\sin 70°)(\cos 70°)$$
$$= -1.58 \text{ MPa} \longleftarrow$$

(b) Largest angle α if $\tau_{allow} = 2.25$ MPa

$$\tau_{allow} = -\sigma_x \sin\theta\cos\theta$$

The shear stress on the joint has a negative sign. Its numerical value cannot exceed $\tau_{allow} = 2.25$ MPa. Therefore,

$-2.25 \text{ MPa} = -(4.9 \text{ MPa})(\sin\theta)(\cos\theta)$ or
$\sin\theta\cos\theta = 0.4592$

From trigonometry: $\sin\theta\cos\theta = \dfrac{1}{2}\sin 2\theta$

Therefore: $\sin 2\theta = 2(0.4592) = 0.9184$

Solving : $2\theta = 66.69°$ or $113.31°$

$\theta = 33.34°$ or $56.66°$

$\alpha = 90° - \theta \quad \therefore \alpha = 56.66°$ or $33.34°$

Since α must be between 10° and 40°, we select

$\alpha = 33.3° \longleftarrow$

Note: If α is between 10° and 33.3°,

$|\tau_\theta| < 2.25$ MPa.

If α is between 33.3° and 40°,

$|\tau_\theta| > 2.25$ MPa.

(c) What is α if $\tau_\theta = 2\sigma_\theta$?

Numerical values only:

$$|\tau_\theta| = \sigma_x \sin\theta\cos\theta \qquad |\sigma_\theta| = \sigma_x \cos^2\theta$$

$$\left|\frac{\tau_\theta}{\sigma_\theta}\right| = 2$$

$$\sigma_x \sin\theta\cos\theta = 2\sigma_x\cos^2\theta$$

$\sin\theta = 2\cos\theta$ or $\tan\theta = 2$

$$\theta = 63.43° \quad \alpha = 90° - \theta$$

$$\alpha = 26.6° \longleftarrow$$

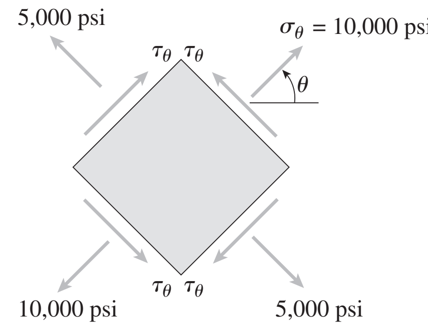
Note: For $\alpha = 26.6°$ and $\theta = 63.4°$, we find $\sigma_\theta = 0.98$ MPa and $\tau_\theta = -1.96$ MPa.

Thus, $\left|\dfrac{\tau_\theta}{\sigma_\theta}\right| = 2$ as required.

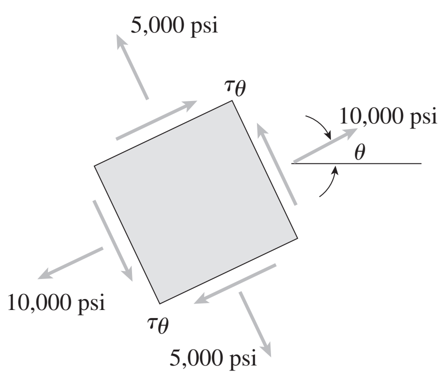
Problem 2.6-15 Acting on the sides of a stress element cut from a bar in uniaxial stress are tensile stresses of 10,000 psi and 5,000 psi, as shown in the figure.

(a) Determine the angle θ and the shear stress τ_θ and show all stresses on a sketch of the element.

(b) Determine the maximum normal stress σ_{max} and the maximum shear stress τ_{max} in the material.



Solution 2.6-15 **Bar in uniaxial stress**



(a) ANGLE θ AND SHEAR STRESS τ_θ

$$\sigma_\theta = \sigma_x \cos^2\theta$$

$$\sigma_\theta = 10{,}000 \text{ psi}$$

$$\sigma_x = \frac{\sigma_\theta}{\cos^2\theta} = \frac{10{,}000 \text{ psi}}{\cos^2\theta} \quad (1)$$

PLANE AT ANGLE $\theta + 90°$

$$\sigma_{\theta+90°} = \sigma_x[\cos(\theta + 90°)]^2 = \sigma_x[-\sin\theta]^2$$

$$= \sigma_x \sin^2\theta$$

$$\sigma_{\theta+90°} = 5{,}000 \text{ psi}$$

$$\sigma_x = \frac{\sigma_{\theta+90°}}{\sin^2\theta} = \frac{5{,}000 \text{ psi}}{\sin^2\theta} \quad (2)$$

Equate (1) and (2):

$$\frac{10{,}000 \text{ psi}}{\cos^2\theta} = \frac{5{,}000 \text{ psi}}{\sin^2\theta}$$

$$\tan^2\theta = \frac{1}{2} \quad \tan\theta = \frac{1}{\sqrt{2}} \quad \theta = 35.26° \longleftarrow$$

From Eq. (1) or (2):

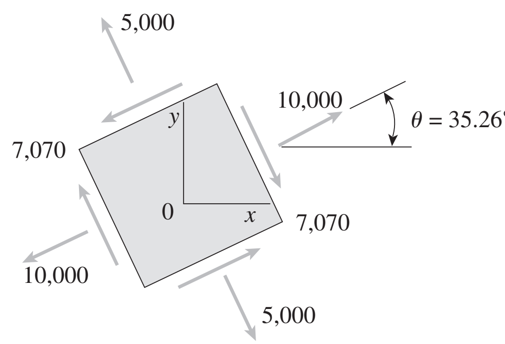
$$\sigma_x = 15{,}000 \text{ psi}$$

$$\tau_\theta = -\sigma_x \sin\theta \cos\theta$$

$$= (-15{,}000 \text{ psi})(\sin 35.26°)(\cos 35.26°)$$

$$= -7{,}070 \text{ psi} \longleftarrow$$

Minus sign means that τ_θ acts clockwise on the plane for which $\theta = 35.26°$.



NOTE: All stresses have units of psi.

(b) MAXIMUM NORMAL AND SHEAR STRESSES

$$\sigma_{max} = \sigma_x = 15{,}000 \text{ psi} \longleftarrow$$

$$\tau_{max} = \frac{\sigma_x}{2} = 7{,}500 \text{ psi} \longleftarrow$$